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SOME DISTANCE MEASURES FOR INTUITIONISTIC UNCERTAIN LINGUISTIC SETS AND THEIR APPLICATION TO GROUP DECISION MAKING

Abstract. *In this paper, we present the intuitionistic uncertain linguistic weighted distance measure. It is a new decision making technique that generalized the OWD measure, having been proved suitable to deal with the situation where the given information is represented in exact numerical values. We investigate the new distance measure in multi-attribute decision making with intuitionistic uncertain linguistic information. Firstly, we develop some distance measures for intuitionistic uncertain linguistic sets, including the intuitionistic uncertain linguistic weighted distance (IULWD) measure, the intuitionistic uncertain linguistic ordered weighted distance (IULOWD) measure, the intuitionistic uncertain linguistic ordered weighted Hamming distance (IULOWHD) measure, the intuitionistic uncertain linguistic ordered weighted Euclidean distance (IULOWED) measure, the intuitionistic uncertain linguistic hybrid weighted distance (IULHWD) measure. These developed distance measures are very suitable to deal with the situation where the input arguments are represented in intuitionistic uncertain linguistic sets. Then we study several desirable properties of the new distance measures and present a consensus reaching process based on the developed distance measures with intuitionistic uncertain linguistic preference information for group decision making. Finally, we apply the developed approach with a numerical example to group decision making under intuitionistic uncertain environment.*

Keywords: *Intuitionistic uncertain linguistic sets; Distance measure;*

Group decision making; Consensus.

JEL Classification: D81, M12, M51

1. Introduction

In the real-life world, distance measure is a common used tool for measuring the deviations of different arguments. Its theory and methods have been widely applied in many fields, such as decision making, medical diagnosis, information fusion, supply chain management and so on. Over the last decades, the study on distance measure has attracted great attentions, refer to [Szmidt and Kacprzyk 2000; Merigó and Gil-Lafuente 2010; Zeng and Su 2012]. Many authors proposed some distance measures including the weighted Hamming distance (WHD) measure and the weighted Euclidean distance (WED) measure, etc. in most of the existing literature; however, these distance measures only consider the importance of each deviation value. To solve the drawback, Xu and Chen (2008) introduced the ordered weighted distance (OWD) measure based on the ordered weighted averaging (OWA) operator (Yager 1988). The prominent characteristic of the OWD measure is that it can relieve (or intensify) the influence of unduly large or small deviations on the aggregation results by assigning low (or high) weights of them and emphasize the importance of the ordered position of the given individual distances instead of weighting arguments themselves. For further research on the other distance measures using the OWA operator and their applications, please see, for example [Merigó and Casanovas 2011; Merigó and Gil-Lafuente 2010; Xu 2007].

The above distance measures just discuss the decision information is expressed in exact numerical numbers. However, in practical applications the available information may be represented by uncertain or fuzzy arguments, including intuitionistic fuzzy sets (IFSs) (Atanassov 1986), interval-value intuitionistic fuzzy sets (IVIFSs) (Atanassov and Gargov 1989), and linguistic labels (Herrera and Martinez 2000) because of time pressure, people's limited expertise related to the problem domain and so on. As a result, Xu (2007) proposed some similarity measures of intuitionistic fuzzy sets based on the geometric distance and the matching function model. Zeng (2013) introduced some intuitionistic fuzzy weighted distance measures, like intuitionistic fuzzy ordered weighted distance (IFOWD) measure and intuitionistic fuzzy hybrid weighted distance (IFHWD) measure. Recently, motivated by the idea of the intuitionistic linguistic set (IUS) (Wang and Li 2009), Liu and Jin (2012) developed the notion of intuitionistic uncertain linguistic set (IULS), which can be viewed as a collection of the intuitionistic uncertain linguistic variables. And then, Liu (2013) proposed an approach to group decision making based on the interval intuitionistic uncertain linguistic sets.

However, in the literature it seems that there is no investigation on distance measure for aggregating a collection of intuitionistic uncertain linguistic sets, except of some similarity measures of intuitionistic fuzzy sets (Xu 2007) and some intuitionistic fuzzy weighted distance measures (Zeng 2013). The research on the consensus reaching process based on distance measures with intuitionistic linguistic preference information for group decision making is in its infancy.

Therefore, based on the intuitionistic uncertain linguistic sets (Liu and Jin 2012), in this paper, we shall develop some distance measures for intuitionistic uncertain linguistic sets, such as the intuitionistic uncertain linguistic weighted distance (IULWD) measure, the intuitionistic uncertain linguistic ordered weighted distance (IULOWD) measure and the intuitionistic uncertain linguistic hybrid weighted distance (IULHWD) measure. These developed distance measures are very suitable to deal with the situations where the available information is represented in intuitionistic uncertain linguistic sets. Also, they can alleviate (or intensify) the influence of unduly large (or small) deviations on the aggregation results by assigning low (or high) weights of them. To do so, this paper is structured as follows. In Section 2, we review some common distance measures and the intuitionistic uncertain linguistic sets. In Section 3, we develop the IULWD measure, the IULOWD measure and the IULHWD measure, and study the various properties of them. In Section 4, we propose an approach to establish a consensus reaching process of intuitionistic uncertain linguistic group decision making. In Section 5, we give a practical application of the developed approach, and the main conclusions of the paper are summarized in Section 6.

2. Preliminaries

In this section, we first review some basic distances measures, and then introduce the notion of the intuitionistic uncertain linguistic sets.

2.1. *Some distance measures*

The weighted Hamming distance (WHD) and the weighted Euclidean distance (WED) are the most wide used distance measures, which are based on the normalized Hamming distance (NHD) and the normalized Euclidean distance (NED) (Kacprzyk 1997).

For two collections of arguments $A=\{a_1,a_2,\cdots,a_n\}$ and $B=\{b_1,b_2,\cdots,b_n\}$, they can be defined as follows:

Definition 1. A weighted Hamming distance (WHD) of dimension *n* is a mapping WHD: $R^n \to R$ that has an associated weighting $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ such that $\omega_j \in [0,1]$, $\sum_{j=1}^{n}\omega_j = 1$ and

$$WHD(A,B)=\sum_{i=1}^{n}\omega_i\left|a_i-b_i\right| \tag{1}$$

where a_i and b_i is the *i*th arguments of the *A* and *B*, respectively.

Definition 2. A weighted Euclidean distance (WED) of dimension *n* is a mapping WED: $R^n \to R$ that has an associated weighting $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ such that $\omega_j \in [0,1]$, $\sum_{j=1}^{n} \omega_j = 1$ and

$$WED(A,B) = \sqrt{\sum_{i=1}^{n} \omega_i \left(a_i - b_i\right)^2} \tag{2}$$

where a_i and b_i is the *i*th arguments of the *A* and *B*, respectively.

Consider a generalization of both the distances measures (1) and (2), a weighted distance (WD) is defined as follows:

$$WD = \left(\sum_{i=1}^{n} \omega_i \left|a_i - b_i\right|^{\lambda}\right)^{1/\lambda}, \ \lambda > 0 \tag{3}$$

However, the above weighted distance measures take only the given individual distances into consideration rather than the ordered positions of the given individual. Yager (1988) developed the wide useful OWA operator, the prominent advantage of the OWA operator is that the input arguments are rearranged in descending order, and the weights associated with the operator are the weights of the ordered positions of the input arguments rather than the weights of the input arguments. Since its appearance, the OWA operator has been widely studied and used in a range of applications, see, for example [Herrera et al. 2003; Peng et al. 2012; Peng and Ye 2013; Xu 2005a]. Motivated by the idea of the OWA operator, Xu and Chen (2008) developed an ordered weighted distance (OWD) measure.

Definition 3. Let $A = \{a_1, a_2, \cdots, a_n\}$ and $B = \{b_1, b_2, \cdots, b_n\}$ be two collections of real numbers, and $d\left(a_j, b_j\right) = \left|a_j - b_j\right|$ be the distance between a_j and b_j, then

$$OWD\left(A,B\right) = \left(\sum_{j=1}^{n} w_j \left(d\left(a_{\sigma(j)}, b_{\sigma(j)}\right)\right)^{\lambda}\right)^{1/\lambda} \tag{4}$$

is called an ordered weighted distance (OWD) between *A* and *B*, in which $\lambda > 0$, $w = (w_1, w_2, \cdots, w_n)^T$ is the weighted vector of the ordered position of the $d\left(a_{\sigma(j)}, b_{\sigma(j)}\right)$, where $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$, and $\left(\sigma(1), \sigma(2), \cdots, \sigma(n)\right)$ is any permutation of $\left(1, 2, \cdots, n\right)$, such that

$$d\left(a_{\sigma(j-1)}, b_{\sigma(j-1)}\right) \geq d\left(a_{\sigma(j)}, b_{\sigma(j)}\right) \tag{5}$$

In the case of $\lambda = 1$ and $\lambda = 2$, the OWD measure is called the ordered weighted Hamming distance (OWHD) measure and the ordered weighted Euclidean distance (OWED) measure:

$$OWHD(A,B)=\sum_{j=1}^{n}w_j d\left(a_{\sigma(j)},b_{\sigma(j)}\right) \tag{6}$$

and

$$OWED(A,B)=\sqrt{\sum_{j=1}^{n}w_j\left(d\left(a_{\sigma(j)},b_{\sigma(j)}\right)\right)^2} \tag{7}$$

Later, Xu (2010) defined a distance measure called intuitionistic fuzzy distance (IFD) based on the concept of intuitionistic fuzzy set (IFS), introduced by Atanassov (1986). The prominent characteristic of IFS is that it assigns to each element a membership degree and a non-membership degree, while the fuzzy set (Zadeh 1965) only assigns to each element a membership degree. Over the last decades, many authors have paid attention to the application to group decision making based on IFS [Atanassov and Gargov 1989; Szmidt and Kacprzyk 2000; Xu 2007; Xu and Yager 2006]. Xu and Yager (2006) introduced the intuitionistic fuzzy numbers (IFNs), which simplify the notion of the IFSs. For any two IFNs, the intuitionistic fuzzy distance (IFD) is defined as follows:

Definition 4. Let α_1 and α_2 be two IFNs, then

$$d\left(\alpha_1,\alpha_2\right)=\frac{1}{2}\left(\left|\mu_{\alpha_1}-\mu_{\alpha_2}\right|+\left|\nu_{\alpha_1}-\nu_{\alpha_2}\right|\right) \tag{8}$$

is called the intuitionistic fuzzy distance (IFD) between α_1 and α_2.

Based on the intuitionistic fuzzy distance (IFD) and the weighted distance (WD), Zeng (2013) proposed some intuitionistic fuzzy weighted distance measures, like intuitionistic fuzzy ordered weighted distance (IFOWD) measure, intuitionistic fuzzy hybrid weighted distance (IFHWD) measure and so on. These weighted distance measures can deal with the situation where the input arguments are represented in intuitionistic fuzzy numbers (IFNs).

Let $S=\left\{s_\alpha \left| \alpha=-t,\cdots,-1,0,1,\cdots,t\right.\right\}$ be a finite and totally ordered discrete term set, where s_α represents a possible value for a linguistic variable. For example, in the case of $t=3$, S can be defined as:

$$S=\{s_{-3}=very\ \ poor, s_{-2}=poor, s_{-1}=slightly\ \ poor, s_0=medium,$$
$$s_1=slightly\ \ good, s_2=good, s_3=very\ \ good\}$$

Note that in the process of given information aggregating, some decision results may do not match any linguistic labels exactly. To preserve all the given information, Xu (2004) extended the discrete label set S to a continuous label set $\overline{S}=\left\{s_\alpha \left| \alpha\in[-q,q]\right.\right\}$, where $q(q>t)$ is a sufficiently large positive number. If $s_\alpha \in S$, we call s_α original linguistic label, otherwise, we call s_α the virtual linguistic label.

With respect to measure the deviation between two linguistic variables $s_\alpha, s_\beta \in \overline{S}$, Xu (2005b) defined the linguistic distance as follows:

Definition 5. Let $s_\alpha, s_\beta \in \overline{S}$, then

$$d(s_\alpha, s_\beta) = |s_\alpha - s_\beta| = \frac{|\alpha - \beta|}{2t} \tag{9}$$

is called the distance measure between s_α and s_β.

2.2. *The intuitionistic uncertain linguistic sets*

Motivated by the idea of the intuitionistic linguistic set (IUS) (Wang and Li 2009) and the uncertain linguistic variables (Xu 2004), Liu and Jin (2012) introduced the notion of intuitionistic uncertain linguistic set (IULS), which can be defined as follows:

Definition 6. An intuitionistic uncertain linguistic set in *X* is given as:

$$A = \{< x[[s_{\alpha(x)}, s_{\beta(x)}], (\mu_A(x), \nu_A(x))] > | x \in X\} \tag{10}$$

where $[s_{\alpha(x)}, s_{\beta(x)}] \in \overline{S}$, $\mu_A(x): X \to [0,1]$ and $\nu_A(x): X \to [0,1]$ with the condition of $0 \le \mu_A(x) + \nu_A(x) \le 1$, $\forall x \in X$. Also, the numbers $\mu_A(x)$ and $\nu_A(x)$ represent the membership degree and non-membership degree of the element *x* to the uncertain linguistic variable $[s_{\alpha(x)}, s_{\beta(x)}]$, respectively. And if $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$, $\forall x \in X$, then $\pi_A(x)$ is called the degree of indeterminacy of the element *x* to the uncertain linguistic variable $[s_{\alpha(x)}, s_{\beta(x)}]$.

For an intuitionistic uncertain linguistic set A, Liu and Jin (2012) defined the intuitionistic uncertain linguistic variable, which can be expressed as the quaternion $< [s_{\alpha(x)}, s_{\beta(x)}], (\mu_A(x), \nu_A(x)) >$. The intuitionistic uncertain linguistic set A can also be viewed as a collection of the intuitionistic uncertain linguistic variables. Therefore, the intuitionistic uncertain linguistic set A can also be denoted by $A = \{< [s_{\alpha(x)}, s_{\beta(x)}], (\mu_A(x), \nu_A(x)) > | x \in X\}$. For any two intuitionistic uncertain linguistic variables α_1 and α_2, the operational laws are defined as follows:

(1) $\alpha_1 \oplus \alpha_2$

$$= < [s_{\alpha(x_1)+\alpha(x_2)}, s_{\beta(x_1)+\beta(x_2)}], (1 - (1 - \mu_A(x_1))(1 - \mu_A(x_2)), \nu_A(x_1)\nu_A(x_2)) >$$

(2) $\alpha_1 \otimes \alpha_2$

$$=<[s_{\alpha(x_1)\times\alpha(x_2)}, s_{\beta(x_1)\times\beta(x_2)}], (\mu_A(x_1)\mu_A(x_2), \nu_A(x_1)+\nu_A(x_2)-\nu_A(x_1)\nu_A(x_2))>$$

(3) $\lambda\alpha_1 =<[s_{\lambda\alpha(x_1)}, s_{\lambda\beta(x_1)}], (1-(1-\mu_A(x_1))^{\lambda}, (\nu_A(x_1))^{\lambda})>$, $\lambda \geq 0$

And the above operational results are still intuitionistic uncertain linguistic variables.

To compare any two intuitionistic uncertain linguistic variables α_1 and α_2, Liu and Jin (2012) proposed a simple method as below:

- If the expected value $E(\alpha_1) < E(\alpha_2)$, then α_1 is smaller than α_2, denoted by $\alpha_1 < \alpha_2$;
- If $E(\alpha_1)=E(\alpha_2)$, then

1) If the accuracy function $H(\alpha_1) < H(\alpha_2)$, then $\alpha_1 < \alpha_2$;
2) If $H(\alpha_1) = H(\alpha_2)$, then α_1 and α_2 represent the same information, denoted by $\alpha_1 = \alpha_2$.

3. Some intuitionistic uncertain linguistic distance measures

In this section, we first define a distance measure for each pair of intuitionistic uncertain linguistic variable, and then develop some intuitionistic uncertain linguistic distance measures. At last, we study the various properties of them.

Definition 7. For any two intuitionistic uncertain linguistic variables $\alpha_1 =<[s_{\alpha(x_1)}, s_{\beta(x_1)}], (\mu(x_1), \nu(x_1))>$ and $\alpha_2 =<[s_{\alpha(x_2)}, s_{\beta(x_2)}], (\mu(x_2), \nu(x_2))>$, then

$$d\left(\alpha_1, \alpha_2\right)$$
$$=\frac{1}{2}(d([s_{\alpha(x_1)}, s_{\beta(x_1)}],[s_{\alpha(x_2)}, s_{\beta(x_2)}])+d((\mu(x_1),\nu(x_1)),(\mu(x_2),\nu(x_2))))$$
$$=\frac{1}{4}((\left|\alpha(x_1)-\alpha(x_2)\right|+\left|\beta(x_1)-\beta(x_2)\right|)/2t+\left|\mu(x_1)-\mu(x_2)\right|+\left|\nu(x_1)-\nu(x_2)\right|) \tag{11}$$

is called the intuitionistic uncertain linguistic distance (IULD) between α_1 and α_2.

Consider two IULSs $A=\{<[s_{\alpha_A(x)}, s_{\beta_A(x)}], (\mu_A(x), \nu_A(x))>\left|x \in X\right.\}$ and $B=\{<[s_{\alpha_B(x)}, s_{\beta_B(x)}], (\mu_B(x), \nu_B(x))>\left|x \in X\right.\}$ on $X=\left\{x_1, x_2, \cdots, x_n\right\}$, we let $A(x)=\alpha$ and $B(x)=\beta$, then the intuitionistic uncertain linguistic sets A and B can be denoted by $A=\left\{\alpha_1, \alpha_2, \cdots, \alpha_n\right\}$ and $B=\left\{\beta_1, \beta_2, \cdots, \beta_n\right\}$. Based on the information above, we can calculate the distance between the intuitionistic uncertain linguistic sets *A* and *B* utilizing the IULD between α_i and β_i , $i=1,2,\cdots,n$.

Definition 8. Let $A=\left(\alpha_1,\alpha_2,\cdots,\alpha_n\right)$ and $B=\left(\beta_1,\beta_2,\cdots,\beta_n\right)$ be two sets of intuitionistic uncertain linguistic variables, then

$$d_{IULWHD}\left(A,B\right)=\sum_{j=1}^{n}\omega_j d_{IULD}(\alpha_j,\beta_j) \tag{12}$$

is called an intuitionistic uncertain linguistic weighted Hamming distance (IULWHD) between *A* and *B*.

Definition 9. Let $A=\left(\alpha_1,\alpha_2,\cdots,\alpha_n\right)$ and $B=\left(\beta_1,\beta_2,\cdots,\beta_n\right)$ be two sets of intuitionistic uncertain linguistic variables, then

$$d_{IULWED}\left(A,B\right)=\sqrt{\sum_{j=1}^{n}\omega_j (d_{IULD}(\alpha_j,\beta_j))^2} \tag{13}$$

is called an intuitionistic uncertain linguistic weighted Euclidean distance (IULWED) between *A* and *B*, where $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ is the weighting vector of the $d_{IULD}(\alpha_j,\beta_j)$ such that $\omega_j\in[0,1]$, $\sum_{j=1}^{n}\omega_j=1$.

Combine Eqs. (12) and (13) to the following form:

$$d_{IULWD}\left(A,B\right)=\left(\sum_{j=1}^{n}\omega_j (d_{IULD}(\alpha_j,\beta_j))^\lambda\right)^{1/\lambda} \tag{14}$$

which is called an intuitionistic uncertain linguistic weighted distance (IULWD) between *A* and *B*. In the case of $\lambda=1$ and $\lambda=2$, the IULWD measure is reduced to the IULWHD measure (12) and the IULWED measure (13), respectively.

Based on the OWD measure (4) and the IULWD measure (14), we can define an intuitionistic uncertain linguistic ordered weighted distance (IULOWD) measure as follows:

Definition 10. Let $A=\left(\alpha_1,\alpha_2,\cdots,\alpha_n\right)$ and $B=\left(\beta_1,\beta_2,\cdots,\beta_n\right)$ be two sets of intuitionistic uncertain linguistic variables, then

$$d_{IULOWD}\left(A,B\right)=\left(\sum_{j=1}^{n}w_j (d_{IULD}(\alpha_{\sigma(j)},\beta_{\sigma(j)}))^\lambda\right)^{1/\lambda} \tag{15}$$

is called an intuitionistic uncertain linguistic ordered weighted distance (IULOWD) between *A* and *B*, where $w=(w_1,w_2,\cdots,w_n)^T$ is the weighted vector of the ordered position of the $d_{IULD}\left(\alpha_{\sigma(j)},\beta_{\sigma(j)}\right)$, with the condition $w_j\in[0,1]$,

$\sum_{j=1}^{n} w_j = 1$. $\left(\sigma(1), \sigma(2), \cdots, \sigma(n)\right)$ is any permutation of $\left(1, 2, \cdots, n\right)$, such that

$$d\left(\alpha_{\sigma(j-1)}, \beta_{\sigma(j-1)}\right) \geq d\left(\alpha_{\sigma(j)}, \beta_{\sigma(j)}\right) \tag{16}$$

Especially, if $\lambda = 1$, then the IULOWD measure is called an intuitionistic uncertain linguistic ordered weighted Hamming distance (IULOWHD) measure:

$$d_{IULOWHD}\left(A, B\right) = \sum_{j=1}^{n} w_j d_{IULD}(\alpha_{\sigma(j)}, \beta_{\sigma(j)}) \tag{17}$$

And if $\lambda = 2$, the IULOWD measure is called an intuitionistic uncertain linguistic ordered weighted Euclidean distance (IULOWED) measure:

$$d_{IULOWED}\left(A, B\right) = \sqrt{\sum_{j=1}^{n} w_j (d_{IULD}(\alpha_{\sigma(j)}, \beta_{\sigma(j)}))^2} \tag{18}$$

From the distance measure Eqs. (14) and (15), we know that the IULWD measure weights the given individual distances while the IULOWD measure weights the ordered positions of the given individual distances instead of weighting the individual distances themselves. Therefore, weights represent different aspects in both the two measures. To overcome the drawback, we develop an intuitionistic uncertain linguistic hybrid weighted distance (IULHWD) measure, which is defined as follows:

Definition 11. Let $A = \left(\alpha_1, \alpha_2, \cdots, \alpha_n\right)$ and $B = \left(\beta_1, \beta_2, \cdots, \beta_n\right)$ be two sets of intuitionistic uncertain linguistic variables, then

$$d_{IULHWD}\left(A, B\right) = \left(\sum_{j=1}^{n} w_j D_{IULD}(\alpha_{\sigma(j)}, \beta_{\sigma(j)})\right)^{1/\lambda} \tag{19}$$

is called an intuitionistic uncertain linguistic hybrid weighted distance (IULHWD) measure between *A* and *B*, where $w = (w_1, w_2, \cdots, w_n)^T$ is the weighting vector associated with the IULHWD measure, $D_{IULD}(\alpha_{\sigma(j)}, \beta_{\sigma(j)})$ is the *j*th largest of the weighted distance $D_{IULD}(\alpha_j, \beta_j)$ ($D_{IULD}(\alpha_j, \beta_j) = n\omega_j \left(d_{IULD}(\alpha_j, \beta_j)\right)^{\lambda}$, $j = 1, 2, \cdots, n$), and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weighting vector of the $d_{IULD}(\alpha_j, \beta_j)$ such that $\omega_j \in [0,1]$, $\sum_{j=1}^{n} \omega_j = 1$, n is the balancing coefficient.

Example 1. Let $A = \left(\alpha_1, \alpha_2, \cdots, \alpha_5\right) = (< [s_{-1}, s_1], (0.6, 0.3) >, < [s_{-4}, s_{-2}],$

$(0.5,0.4)>,<[s_2,s_3],(0.7,0.1)>,<[s_0,s_1],(0.4,0.5)>,<[s_3,s_4],(0.2,0.6)>)$ and $B=(\beta_1,\beta_2,\cdots,\beta_5)=(<[s_{-3},s_{-2}],(0.5,0.4)>,<[s_3,s_4],(0.7,0.3)>,$ $<[s_0,s_2],(0.2,0.5)>,<[s_3,s_4],(0.4,0.1)>,<[s_1,s_2],(0.8,0.2)>$ be two sets of intuitionistic uncertain linguistic variables, then by utilizing Eq. (11), we can get

$$d_{IULD}(\alpha_1,\beta_1)=0.20625,\quad d_{IULD}(\alpha_2,\beta_2)=0.48125,$$
$$d_{IULD}(\alpha_3,\beta_3)=0.31875,\quad d_{IULD}(\alpha_4,\beta_4)=0.2875,$$
$$d_{IULD}(\alpha_5,\beta_5)=0.375$$

Suppose that $\omega=(0.15,0.3,0.1,0.25,0.2)$, and without loss of generality, let $\lambda=2$, then we can get

$$D_{IULD}(\alpha_1,\beta_1)=5\times0.15\times0.20625^2=0.0319$$

Similarly, we have

$$D_{IULD}(\alpha_2,\beta_2)=0.3474,\quad D_{IULD}(\alpha_3,\beta_3)=0.0508,$$
$$D_{IULD}(\alpha_4,\beta_4)=0.1033,\quad D_{IULD}(\alpha_5,\beta_5)=0.1406$$

The weight vector associated with the intuitionistic uncertain linguistic hybrid weighted distance (IULHWD) measure $w=(0.11,0.24,0.3,0.24,0.11)^T$, which is derived by using the Gaussian distribution based method, for more details, refer to Xu (2005a). Then we can get the IULHWD between *A* and *B* by utilizing Eq. (19):

$$\begin{aligned}d_{IULHWD}(A,B)&=\left(\sum_{j=1}^{5}w_jD_{IULD}(\alpha_{\sigma(j)},\beta_{\sigma(j)})\right)^{1/2}\\&=(0.11\times0.3474+0.24\times0.1406+0.3\times0.1033+0.24\times0.0508\\&\quad+0.11\times0.0319)^{1/2}\\&=0.3445\end{aligned}$$

Theorem 1 The IULWD measure is a special case of the IULHWD measure.

Proof. Let $w=(\frac{1}{n},\frac{1}{n},\cdots,\frac{1}{n})^T$, then

$$\begin{aligned}d_{IULHWD}(A,B)&=\left(\sum_{j=1}^{n}w_jD_{IULD}(\alpha_{\sigma(j)},\beta_{\sigma(j)})\right)^{1/\lambda}=\left(\frac{1}{n}\sum_{j=1}^{n}D_{IULD}(\alpha_j,\beta_j)\right)^{1/\lambda}\\&=\left(\frac{1}{n}\sum_{j=1}^{n}n\omega_j(d_{IULD}(\alpha_j,\beta_j))^{\lambda}\right)^{1/\lambda}=\left(\sum_{j=1}^{n}\omega_j(d_{IULD}(\alpha_j,\beta_j))^{\lambda}\right)^{1/\lambda}\\&=d_{IULWD}(A,B)\end{aligned}$$

which completes the proof of Theorem 1.

Theorem 2. The IULOWD measure is a special case of the IULHWD measure.

Proof. Let $\omega = (\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n})^T$, then

$$D_{IULD}(\alpha_{\sigma(j)}, \beta_{\sigma(j)}) = n\omega_j (d_{IULD}(\alpha_{\sigma(j)}, \beta_{\sigma(j)}))^{\lambda} = (d_{IULD}(\alpha_{\sigma(j)}, \beta_{\sigma(j)}))^{\lambda}$$

which completes the proof of Theorem 2.

From the IULHWD measure and above analysis, we can get:

(1) By computational analysis, we know that the IULHWD measure can relieve (or intensify) the influence of unduly large or small difference individual on the aggregation results by assigning them low (or high) weights.

(2) The IULHWD measure generalizes both the IULWD and IULOWD measure and reflects the importance degrees of both the given individual distances and their ordered positions.

(3) In fact, firstly, the IULHWD measure weights the given individual distances, and then reorders the weighted individual distances in descending order and weights these ordered individual distances by the IULHWD weights. At last, we process these individual distances into a collective one under the parameter λ.

4. An approach to group decision making based on intuitionistic uncertain linguistic variables

Consider a group decision making problem with intuitionistic uncertain linguistic information. Let $X = \{x_1, x_2, \cdots, x_n\}$ be a discrete set of alternatives, $d_k \in D (k = 1, 2, \cdots, m)$ be the set of decision makers (DMs), and $u = (u_1, u_2, \cdots, u_m)^T$ be the weight vector of DMs, with the condition $u_k \geq 0$, $\sum_{k=1}^{m} u_k = 1$. The DMs $d_k (k = 1, 2, \cdots, m)$ provide their preferences with intuitionistic uncertain linguistic variable $\alpha_{kj} (j = 1, 2, \cdots, n)$ over all the alternatives $x_j \in X$ respect to a criterion. For computational convenience, the preference vectors of all the DMs d_k are denoted by:

$$\alpha_k = (\alpha_{k1}, \alpha_{k2}, \cdots, \alpha_{kn}), \quad k = 1, 2, \cdots, m \qquad (20)$$

Based on the above decision information, we shall develop an approach to reaching consensus of group opinions utilize the IULHWD measure as follows:

Step 1: Calculate the collective preference vector $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n)$ by using the intuitionistic uncertain linguistic weighted average operator, and we have

$$\alpha_j = u_1\alpha_{1j} + u_2\alpha_{2j} + \cdots + u_m\alpha_{mj}, \quad j = 1, 2, \cdots, n \tag{21}$$

Step 2: Calculate the distance $d_{IULD}(\alpha_{kj}, \alpha_j)$ of each preference value α_{kj} given by the decision maker d_k and the corresponding collective preference with intuitionistic uncertain linguistic variable α_j by using Eq. (11).

Step 3: By using Eq. (19), we calculate the IULHWD measure between the preference vectors α_k and α:

$$d_{IULHWD}\left(\alpha_k, \alpha\right) = \left(\sum_{j=1}^{n} w_j D_{IULD}(\alpha_{\sigma(kj)}, \alpha_{\sigma(j)})\right)^{1/\lambda} \tag{22}$$

where $w_j = (w_1, w_2, \cdots, w_n)^T$ is the weighting vector associated with the IULHWD measure, can be derived by using the Gaussian distribution based method (Xu 2005a), $D_{IULD}(\alpha_{\sigma(kj)}, \alpha_{\sigma(j)})$ is the *j*th largest of the weighted distance $D_{IULD}(\alpha_{kj}, \alpha_j)$ ($D_{IULD}(\alpha_{kj}, \alpha_j) = n\omega_j \left(d_{IULD}(\alpha_{kj}, \alpha_j)\right)^{\lambda}$, $j = 1, 2, \cdots, n$), and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weighting vector of the $d_{IULD}(\alpha_{kj}, \alpha_j)$ such that $\omega_j \in [0,1]$, $\sum_{j=1}^{n} \omega_j = 1$.

Step 4: A discussion on the consensus reaching process for group decision making:

(1) Let ρ be the threshold value of acceptable consensus, if all $d_{IULHWD}\left(\alpha_k, \alpha\right) \le \rho \quad (k = 1, 2, \cdots, m)$, then the group is of acceptable consensus. Therefore, it can be determined by the group in practical applications.

(2) Otherwise, if there exists some k_0, such that $d_{IULHWD}\left(\alpha_{k_0}, \alpha\right) > \rho$, then we shall return α_{k_0} (together with α as a reference) to the decision maker d_k for revaluation, and repeat this consensus reaching process until $d_{IULHWD}\left(\alpha_{k_0}, \alpha\right) \le \rho$ or the number of rounds reach the maximum which is predefined by the group so as to avoid stagnation.

5. Illustrative example

Let us consider a decision making problem of evaluating port logistics system for vulnerability and promotion discussed in (Zhang et al. 2011). Among many criterions of the system evaluation, “cargo throughput” is the main criterion used.

There are four port candidates $x_j \in X\,(j=1,2,3,4)$ and three decision makers (DMs) $d_k \in D(k=1,2,3)$ (whose weighting vector is $u=(0.4,0.3,0.3)^T$). And each decision maker d_k provides his/her preferences with intuitionistic uncertain linguistic variable $\alpha_{kj}(k=1,2,3;\,j=1,2,3,4)$ over all the port candidates x_j, shown in Table 1.

Table 1. Decision matrix with IULSs

	x_1	x_2	x_3	x_4
d_1	$<[s_3,s_4],(0.3,0.6)>$	$<[s_0,s_1],(0.7,0.2)>$	$<[s_1,s_2],(0.4,0.6)>$	$<[s_0,s_1],(0.4,0.3)>$
d_2	$<[s_1,s_1],(0.4,0.3)>$	$<[s_{-1},s_0],(0.8,0.2)>$	$<[s_{-4},s_{-2}],(0.6,0.1)>$	$<[s_2,s_3],(0.2,0.8)>$
d_3	$<[s_2,s_3],(0.1,0.3)>$	$<[s_{-1},s_1],(0.7,0.1)>$	$<[s_1,s_2],(0.4,0.5)>$	$<[s_{-3},s_{-2}],(0.3,0.4)>$

For computational convenience, the preferences of all the DMs $d_k \in D(k=1,2,3)$ are denoted by the vector forms as follows:

$$\alpha_1=(\alpha_{11},\alpha_{12},\alpha_{13},\alpha_{14})=(<[s_3,s_4],(0.3,0.6)>,<[s_0,s_1],(0.7,0.2)>,\\ <[s_1,s_2],(0.4,0.6)>,<[s_0,s_1],(0.4,0.3)>)$$

$$\alpha_2=(\alpha_{21},\alpha_{22},\alpha_{23},\alpha_{24})=(<[s_1,s_1],(0.4,0.3)>,<[s_{-1},s_0],(0.8,0.2)>,\\ <[s_{-4},s_{-2}],(0.6,0.1)>,<[s_2,s_3],(0.2,0.8)>)$$

$$\alpha_3=(\alpha_{31},\alpha_{32},\alpha_{33},\alpha_{34})=(<[s_2,s_3],(0.1,0.3)>,<[s_{-1},s_1],(0.7,0.1)>,\\ <[s_1,s_2],(0.4,0.5)>,<[s_{-3},s_{-2}],(0.3,0.4)>)$$

Calculate the collective preference vector by using

$$\alpha_j=u_1\alpha_{1j}+u_2\alpha_{2j}+u_3\alpha_{3j},\quad j=1,2,3,4$$

We can have

$$\alpha=(\alpha_1,\alpha_2,\alpha_3,\alpha_4)\\ =(<[s_{2.1},s_{2.8}],(0.3,0.4)>,<[s_{-0.6},s_{0.7}],(0.7,0.2)>,\\ <[s_{-0.5},s_{0.8}],(0.5,0.3)>,<[s_{-0.3},s_{0.7}],(0.4,0.5)>)$$

And then, by using Eq. (11), we can calculate the distance $d_{IULD}(\alpha_{kj},\alpha_j)$ of each preference value α_{kj} given by the DM d_k and the corresponding collective preference with intuitionistic uncertain linguistic variable α_j:

$d_{IULD}(\alpha_{11},\alpha_1)=0.1156$, $d_{IULD}(\alpha_{12},\alpha_2)=0.128$, $d_{IULD}(\alpha_{13},\alpha_3)=0.1844$, $d_{IULD}(\alpha_{14},\alpha_4)=0.0688$; $d_{IULD}(\alpha_{21},\alpha_1)=0.14$, $d_{IULD}(\alpha_{22},\alpha_2)=0.1344$, $d_{IULD}(\alpha_{23},\alpha_3)=0.053$, $d_{IULD}(\alpha_{24},\alpha_4)=0.268$; $d_{IULD}(\alpha_{31},\alpha_1)=0.0844$, $d_{IULD}(\alpha_{32},\alpha_2)=0.1219$, $d_{IULD}(\alpha_{33},\alpha_3)=0.16$, $d_{IULD}(\alpha_{34},\alpha_4)=0.1187$.

Without loss of generality, let $\lambda=1$ and $\omega=(0.2,0.35,0.15,0.3)^T$, the weight vector associated with the intuitionistic uncertain linguistic hybrid weighted distance measure $w=(0.155,0.345,0.345,0.155)^T$, which is derived by the Gaussian distribution based method (Xu (2005a)). Then we calculate the IULHWD measure between the preference vectors α_k and α by using Eq. (22):

$$d_{IULHWD}\left(\alpha_1,\alpha\right)=0.112,\ d_{IULHWD}\left(\alpha_2,\alpha\right)=0.178,\ d_{IULHWD}\left(\alpha_3,\alpha\right)=0.120.$$

Suppose the threshold value of acceptable consensus is $\rho=0.150$, then we can get

$$d_{IULHWD}\left(\alpha_2,\alpha\right)>0.150$$

Now, we need to return α_2 (together with α as a reference) to the DM d_2 for revaluation. Suppose the revaluated α_2 is

$$\begin{aligned}\alpha_2=(\alpha_{21},\alpha_{22},\alpha_{23},\alpha_{24})=(&<[s_0,s_1],(0.4,0.2)>,<[s_{-1},s_0],(0.7,0.2)>,\\&<[s_{-3},s_{-2}],(0.5,0.2)>,<[s_2,s_3],(0.2,0.6)>)\end{aligned}$$

And we can calculate the collective preference vector once more

$$\begin{aligned}\alpha&=(\alpha_1,\alpha_2,\alpha_3,\alpha_4)\\&=(<[s_{1.8},s_{2.8}],(0.3,0.3)>,<[s_{-0.6},s_{0.7}],(0.6,0.2)>,\\&\quad<[s_{-0.2},s_{0.8}],(0.3,0.4)>,<[s_{-0.3},s_{0.7}],(0.4,0.4)>)\end{aligned}$$

Respectively, we can have the IULHWD measure between the preference vectors α_k and α by using Eqs. (11) and (22) (let $\lambda=1$):

$$d_{IULHWD}\left(\alpha_1,\alpha\right)=0.088,\ d_{IULHWD}\left(\alpha_2,\alpha\right)=0.146,\ d_{IULHWD}\left(\alpha_3,\alpha\right)=0.103.$$

As we can see, the recalculated numerical results are less than 0.150, that is $d_{IULHWD}\left(\alpha_k,\alpha\right)\le 0.150$ $(k=1,2,3)$. Thus, all the distances are less than the predefined threshold value of acceptable consensus, which indicates that the group reaches consensus. Moreover, the process of group reaches consensus in the cases of $\lambda=2$ can be discussed similar to the case of $\lambda=1$.

6. Conclusions

In this paper, we have suggested several extensions of the common used distance measures when dealing with uncertain linguistic situations. We first introduce the notion of intuitionistic uncertain linguistic sets and intuitionisic uncertain linguistic variables, and then develop some new distance measures to accommodate the situations where the given arguments are intuitionistic uncertain linguistic information, including the IULWD measure, the IULOWD measure, and the IULHWD measure and so on. Also, we have studied several desirable properties of the IULHWD measure, which can alleviate the influence of unduly large (or small) deviations on the aggregation results by assigning them low (or high) weights. We have presented a new group decision making process based on the IULHWD measure and finally we have focused on an application in a group decision making problem of evaluating port logistics system for vulnerability and promotion.

In the future, we shall continue working in extending the distance measures to deal with the situations where the input arguments are expressed in other uncertain information including interval intuitionistic uncertain vaviables, triangular intuitionistic fuzzy values or pure linguistic labels. We will also investigate different types of applications in other domains based on the distance measures.

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